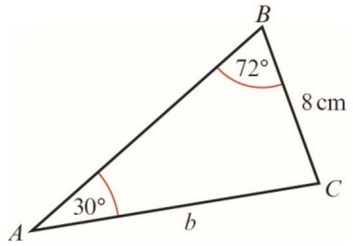
Exercise 6B

1 a



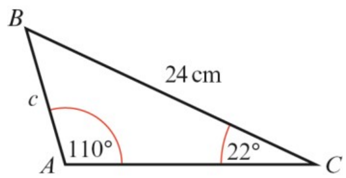
Using $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$\dfrac{8}{\sin 30°}=\dfrac{b}{\sin 72°}$

$\Rightarrow b=\dfrac{8\sin 72°}{\sin 30°}=15.2\text{ cm}\ (3\text{ s.f.})$

$(\text{As } 72°>30°,\ b>8\text{ cm})$

b



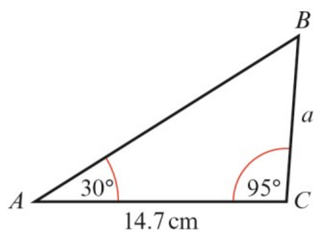
Using $\dfrac{a}{\sin A}=\dfrac{c}{\sin C}$

$\dfrac{24}{\sin 110°}=\dfrac{c}{\sin 22°}$

$\Rightarrow c=\dfrac{24\sin 22°}{\sin 110°}=9.57\text{ cm}\ (3\text{ s.f.})$

$(\text{As } 110°>22°,\ 24\text{ cm}>c.)$

c



$\angle ABC=180°-(30+95)°$

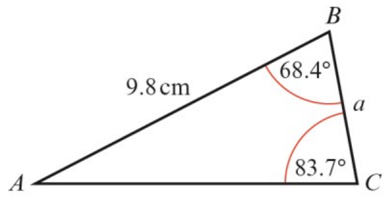
$=55°$

Using $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$\dfrac{a}{\sin 30°}=\dfrac{14.7}{\sin 55°}$

$\Rightarrow a=\dfrac{14.7\sin 30°}{\sin 55°}=8.97\text{ cm}\ (3\text{ s.f.})$

d



$\angle ABC=180°-(68.4+83.7)°$

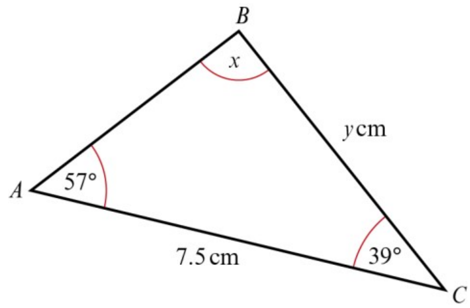
$=27.9°$

Using $\dfrac{a}{\sin A}=\dfrac{c}{\sin C}$

$\dfrac{a}{\sin 27.9°}=\dfrac{9.8}{\sin 83.7°}$

$\Rightarrow a=\dfrac{9.8\sin 27.9°}{\sin 83.7°}=4.61\text{ cm}\ (3\text{ s.f.})$

2 a



$x=180°-(57+39)°$

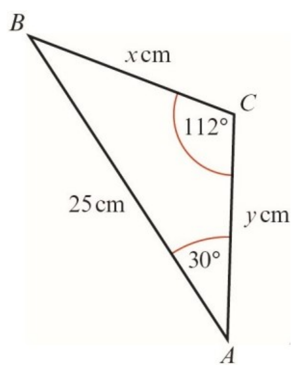
$=84°$

Using $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$\dfrac{y}{\sin 57°}=\dfrac{7.5}{\sin 84°}$

$\Rightarrow y=\dfrac{7.5\sin 57°}{\sin 84°}=6.32\text{ cm}\ (3\text{ s.f.})$

b



Using $\dfrac{a}{\sin A}=\dfrac{c}{\sin C}$

2 **b** $\dfrac{x}{\sin 30°} = \dfrac{25}{\sin 112°}$

$\Rightarrow x = \dfrac{25 \sin 30°}{\sin 112°} = 13.5 \text{ cm } (3 \text{ s.f.})$

$\angle B = 180° - (112 + 30)°$

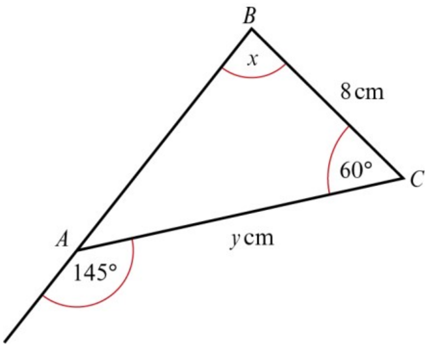
$= 38°$

Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{y}{\sin 38°} = \dfrac{25}{\sin 112°}$

$\Rightarrow y = \dfrac{25 \sin 38°}{\sin 112°} = 16.6 \text{ cm } (3 \text{ s.f.})$

c



$x = 180° - (60 + 35)°$

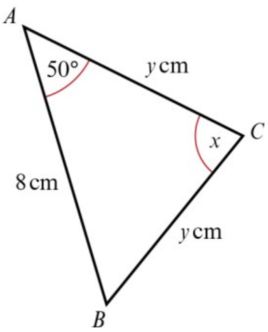
$= 85°$

Using $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{y}{\sin 85°} = \dfrac{8}{\sin 35°}$

$\Rightarrow y = \dfrac{8 \sin 85°}{\sin 35°} = 13.9 \text{ cm } (3 \text{ s.f.})$

d



$x = 180° - (50 + 50)°$

$= 80°$

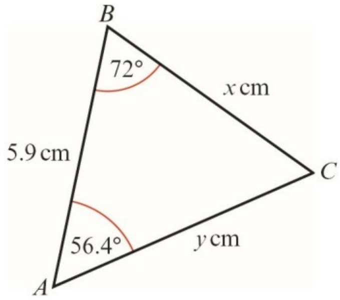
Using $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

d $\dfrac{y}{\sin 50°} = \dfrac{8}{\sin 80°}$

$\Rightarrow y = \dfrac{8 \sin 50°}{\sin 80°} = 6.22 \text{ cm } (3 \text{ s.f.})$

(Note: You could use the line of symmetry to split the triangle into two right-angled triangles and use $\cos 50° = \dfrac{4}{y}$.)

e



$\angle C = 180° - (56.4 + 72)°$

$= 51.6°$

Using $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{x}{\sin 56.4°} = \dfrac{5.9}{\sin 51.6°}$

$\Rightarrow x = \dfrac{5.9 \sin 56.4°}{\sin 51.6°}$

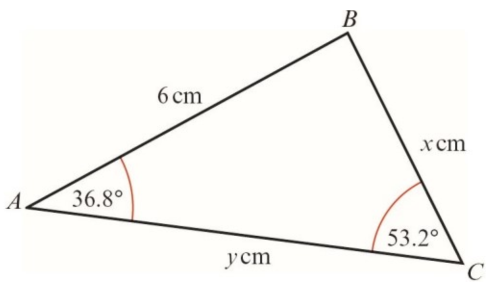
$= 6.27 \text{ cm } (3 \text{ s.f.})$

Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{y}{\sin 72°} = \dfrac{5.9}{\sin 51.6°}$

$\Rightarrow y = \dfrac{5.9 \sin 72°}{\sin 51.6°} = 7.16 \text{ cm } (3 \text{ s.f.})$

f



Using $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{x}{\sin 36.8°} = \dfrac{6}{\sin 53.2°}$

2 f $\Rightarrow x = \dfrac{6\sin 36.8°}{\sin 53.2°} = 4.49 \text{ cm} \left(3 \text{ s.f.}\right)$

$\angle B = 180° - \left(36.8 + 53.2\right)°$

$= 90°$

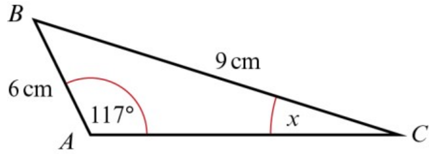
Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{6}{\sin 53.2°} = \dfrac{y}{\sin 90°}$

$\Rightarrow y = \dfrac{6\sin 90°}{\sin 53.2°} = 7.49 \text{ cm} \left(3 \text{ s.f.}\right)$

(Note: The third angle is 90° so you could solve the problem using sine or cosine; the sine rule is not necessary.)

3 a



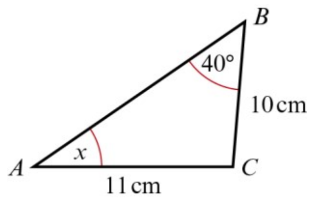
Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin x}{6} = \dfrac{\sin 117°}{9}$

$\Rightarrow \sin x = \dfrac{6\sin 117°}{9} \left(= 0.5940\ldots\right.$

$x = \sin^{-1}\left(\dfrac{6\sin 117°}{9}\right) = 36.4° \left(3 \text{ s.f.}\right)$

b



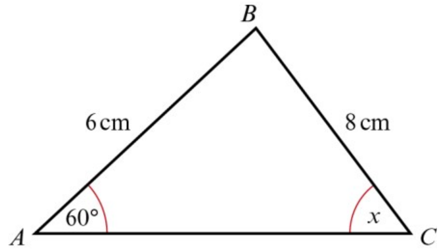
Using $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\dfrac{\sin x}{10} = \dfrac{\sin 40°}{11}$

$\Rightarrow \sin x = \dfrac{10\sin 40°}{11} (= 0.5843\ldots$

$x = \sin^{-1}\left(\dfrac{10\sin 40°}{11}\right) = 35.8° \text{ (3 s.f.)}$

3 c



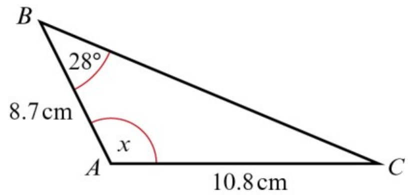
Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin x}{6} = \dfrac{\sin 60°}{8}$

$\Rightarrow \sin x = \dfrac{6\sin 60°}{8} \left(= 0.6495\ldots\right.$

$x = \sin^{-1}\left(\dfrac{6\sin 60°}{8}\right) = 40.5° \left(3 \text{ s.f.}\right)$

d



Using $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{8.7} = \dfrac{\sin 28°}{10.8}$

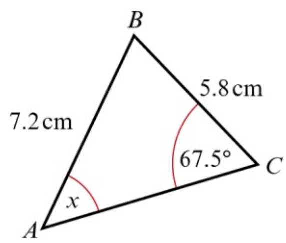
$\Rightarrow \sin C = \dfrac{8.7\sin 28°}{10.8} \left(= 0.3781\ldots\right.$

$C = \sin^{-1}\left(\dfrac{8.7\sin 28°}{10.8}\right)$

$C = 22.2° \text{ (3 s.f.)}$

$\Rightarrow x = 180° - (28 + 22.2)° = 130° \text{ (3 s.f.)}$

4 a



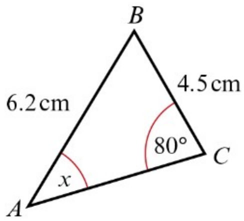
Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x}{5.8} = \dfrac{\sin 67.5°}{7.2}$

4 a $\Rightarrow \sin x = \dfrac{5.8\sin 67.5°}{7.2} (= 0.7442\ldots$

$\Rightarrow x = \sin^{-1}\left(\dfrac{5.8\sin 67.5°}{7.2}\right) = 48.1° \text{ (3 s.f.)}$

b



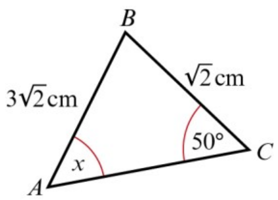
Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x}{4.5} = \dfrac{\sin 80°}{6.2}$

$\Rightarrow \sin x = \dfrac{4.5\sin 80°}{6.2} (= 0.7147\ldots$

$\Rightarrow x = \sin^{-1}\left(\dfrac{4.5\sin 80°}{6.2}\right) = 45.6° \text{ (3 s.f.)}$

c



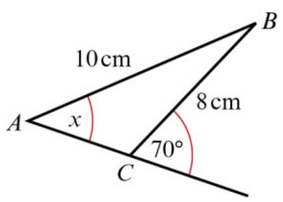
Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x}{\sqrt{2}} = \dfrac{\sin 50°}{3\sqrt{2}}$

$\Rightarrow \sin x = \dfrac{\sqrt{2}\sin 50°}{3\sqrt{2}} (= 0.2553\ldots$

$\Rightarrow x = \sin^{-1}\left(\dfrac{\sin 50°}{3}\right) = 14.8° \text{ (3 s.f.)}$

d



d Angle $ACB = 180° - 70° = 110°$

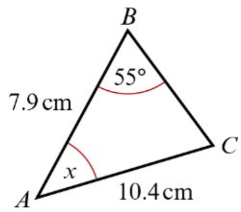
Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x}{8} = \dfrac{\sin 110°}{10}$

$\Rightarrow \sin x = \dfrac{8\sin 110°}{10} (= 0.7517\ldots$

$x = \sin^{-1}\left(\dfrac{8\sin 110°}{10}\right) = 48.7° \text{ (3 s.f.)}$

e



Using $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{7.9} = \dfrac{\sin 55°}{10.4}$

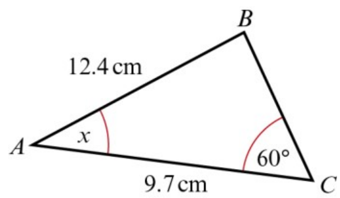
$\Rightarrow \sin C = \dfrac{7.9\sin 55°}{10.4}$

$C = \sin^{-1}\left(\dfrac{7.9\sin 55°}{10.4}\right) = 38.48°$

$x = 180° - (55° + C)$

$\Rightarrow x = 86.52° = 86.5° \text{ (3 s.f.)}$

f



Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin B}{9.7} = \dfrac{\sin 60°}{12.4}$

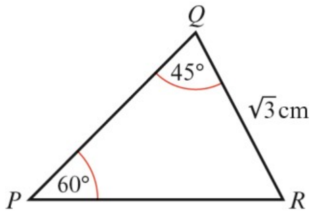
$\Rightarrow \sin B = \dfrac{9.7\sin 60°}{12.4} (= 0.6774\ldots$

$B = 42.65°$

$x = 180° - (60 + B)° = 77.35°$

$\Rightarrow x = 77.4° \text{ (3 s.f.)}$

5



a Using $\dfrac{q}{\sin Q} = \dfrac{p}{\sin P}$

$$\frac{PR}{\sin 45^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow PR = \frac{\sqrt{3}\sin 45^\circ}{\sin 60^\circ} = 1.41 \text{ cm } (3 \text{ s.f.})$$

(The exact answer is $\sqrt{2}$ cm.)

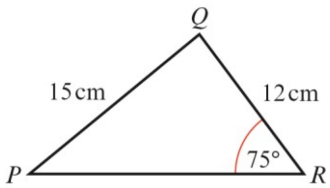
b Using $\dfrac{r}{\sin R} = \dfrac{p}{\sin P}$

$(R = 180^\circ - (60+45)^\circ = 75^\circ)$

$$\frac{PQ}{\sin 75^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow PQ = \frac{\sqrt{3}\sin 75^\circ}{\sin 60^\circ} = 1.93 \text{ cm } (3 \text{ s.f.})$$

6



Using $\dfrac{\sin P}{p} = \dfrac{\sin R}{r}$

$$\frac{\sin P}{12} = \frac{\sin 75^\circ}{15}$$

$$\Rightarrow \sin P = \frac{12\sin 75^\circ}{15}$$

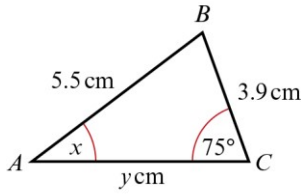
$$P = \sin^{-1}\left(\frac{12\sin 75^\circ}{15}\right) = 50.60^\circ$$

Angle $QPR = 50.6^\circ$ (3 s.f.)

Angle $PQR = 180^\circ - (75 + 50.6)^\circ$

$= 54.4^\circ$ (3 s.f.)

7 a



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin x}{3.9} = \frac{\sin 75^\circ}{5.5}$$

$$\Rightarrow \sin x = \frac{3.9\sin 75^\circ}{5.5}$$

$$x = \sin^{-1}\left(\frac{3.9\sin 75^\circ}{5.5}\right) = 43.23^\circ$$

$\Rightarrow x = 43.2^\circ$ (3 s.f.)

So $\angle ABC = 180^\circ - (75 + 43.2)^\circ = 61.8^\circ$

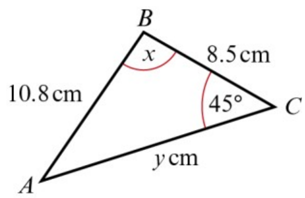
Using $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\frac{y}{\sin 61.8^\circ} = \frac{5.5}{\sin 75^\circ}$$

$$\Rightarrow y = \frac{5.5\sin 61.8^\circ}{\sin 75^\circ} = 5.018$$

$y = 5.02$ cm (3 s.f.)

b



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\frac{\sin A}{8.5} = \frac{\sin 45^\circ}{10.8}$$

$$\Rightarrow \sin A = \frac{8.5\sin 45^\circ}{10.8}$$

$$A = \sin^{-1}\left(\frac{8.5\sin 45^\circ}{10.8}\right) = 33.815^\circ$$

$$x = 180^\circ - (45^\circ + A) = 101.2^\circ$$

$\Rightarrow x = 101^\circ$ (3 s.f.)

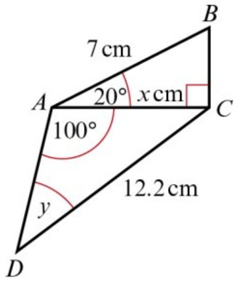
7 b Using $\frac{b}{\sin B}=\frac{c}{\sin C}$

$$\frac{y}{\sin x}=\frac{10.8}{\sin 45°}$$

$$\Rightarrow y=\frac{10.8\sin x}{\sin 45°}=14.98°$$

$$y=15.0\text{ cm }(3\text{ s.f.})$$

c



In ΔABC, $\frac{x}{7}=\cos 20°$

$$\Rightarrow x=7\cos 20°$$

$$=6.58\text{ cm }(3\text{ s.f.})$$

Using $\frac{\sin D}{d}=\frac{\sin A}{a}$ in ΔADC

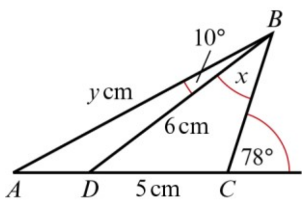
$$\frac{\sin y}{x}=\frac{\sin 100°}{12.2}$$

$$\Rightarrow \sin y=\frac{x\sin 100°}{12.2}$$

$$y=\sin^{-1}\left(\frac{x\sin 100°}{12.2}\right)=32.07°$$

$$\Rightarrow y=32.1°\ (3\text{ s.f.})$$

d



In triangle BDC:

$\angle C=180°-78°=102°$

Using $\frac{\sin B}{b}=\frac{\sin C}{c}$

$$\frac{\sin x}{5}=\frac{\sin 102°}{6}$$

$$\Rightarrow \sin x=\frac{5\sin 102°}{6}$$

d $x=\sin^{-1}\left(\frac{5\sin 102°}{6}\right)=54.599°$

$$\Rightarrow x=54.6°\ (3\text{ s.f.})$$

In triangle ABC:

$\angle BAC=180°-102°-(10+x)°=13.4°$

So $\angle ADB=180°-10°-13.4°=156.6°$

Using $\frac{d}{\sin D}=\frac{a}{\sin A}$ in ΔABD

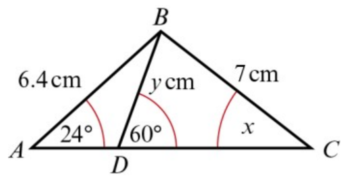
$$\frac{y}{\sin 156.6°}=\frac{6}{\sin 13.4°}$$

$$\Rightarrow y=\frac{6\sin 156.6°}{\sin 13.4°}$$

$$=10.28$$

$$=10.3\text{ cm }(3\text{ s.f.})$$

e



Using $\frac{\sin C}{c}=\frac{\sin A}{a}$ in ΔABC

$$\frac{\sin x}{6.4}=\frac{\sin 24°}{7}$$

$$\Rightarrow x=21.8°\ (3\text{ s.f.})$$

Using $\frac{a}{\sin A}=\frac{d}{\sin D}$ in ΔABD

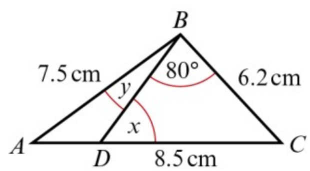
$$\frac{y}{\sin 24°}=\frac{6.4}{\sin 120°}$$

$$\Rightarrow y=\frac{6.4\sin 24°}{\sin 120°}=3.0058$$

$$y=3.01\text{ cm }(3\text{ s.f.})$$

(The above approach finds the two values independently. You could find y first and then use it to find x, but if your answer for y is wrong then x will be wrong as well.)

f



7 f Using $\frac{\sin D}{d}=\frac{\sin B}{b}$ in ΔBDC

$$\frac{\sin x}{6.2}=\frac{\sin 80°}{8.5}$$

$$\Rightarrow \sin x=\frac{6.2\sin 80°}{8.5}$$

$$x=\sin^{-1}\left(\frac{6.2\sin 80°}{8.5}\right)=45.92°$$

$$\Rightarrow x=45.9\ (3\text{ s.f.})$$

In triangle ABC:

$$\angle ACB=180°-(80+x)°$$
$$=54.08°$$

Using $\frac{\sin A}{a}=\frac{\sin C}{c}$

$$\frac{\sin A}{6.2}=\frac{\sin 54.08°}{7.5}$$

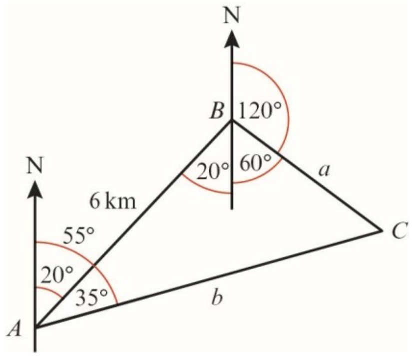
$$\Rightarrow \sin A=\frac{6.2\sin 54.08°}{7.5}$$

$$A=\sin^{-1}\left(\frac{6.2\sin 54.08°}{7.5}\right)=42.03°$$

So $y°=180°-(42.03+134.1)°$

$y=3.87°\ (3\text{ s.f.})$

8



$\angle BAC=55°-20°=35°$

$\angle ABC=20°+60°=80°$

(Alternate angles and angles on a straight line.)

$\angle ACB=180°-(80+35)°=65°$

a Using $\frac{b}{\sin B}=\frac{c}{\sin C}$

$$\Rightarrow \frac{AC}{\sin 80°}=\frac{6}{\sin 65°}$$

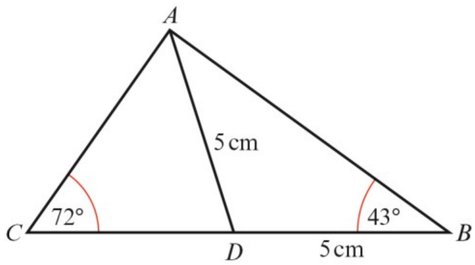
$$\Rightarrow AC=\frac{6\sin 80°}{\sin 65°}=6.52\text{ km}\ (3\text{ s.f.})$$

8 b Using $\frac{a}{\sin A}=\frac{c}{\sin C}$

$$\Rightarrow \frac{BC}{\sin 35°}=\frac{6}{\sin 65°}$$

$$\Rightarrow BC=\frac{6\sin 35°}{\sin 65°}=3.80\text{ km}\ (3\text{ s.f.})$$

9



a In triangle ABD:

$\angle DAB=43°$ (isosceles Δ)

So $\angle ADB=180°-(2\times 43°)=94°$

As the triangle is isosceles you could work with right-angled triangles, but using the sine rule:

$$\frac{d}{\sin D}=\frac{a}{\sin A}$$

$$\Rightarrow \frac{AB}{\sin 94°}=\frac{5}{\sin 43°}$$

$$AB=\frac{5\sin 94°}{\sin 43°}=7.31\text{cm}\ (3\text{ s.f.})$$

b In triangle ADC:

$\angle ADC=180°-94°=86°$

So $\angle CAD=180°-(72+86)°=22°$

Using $\frac{a}{\sin A}=\frac{c}{\sin C}$

$$\Rightarrow \frac{CD}{\sin 22°}=\frac{5}{\sin 72°}$$

$$CD=\frac{5\sin 22°}{\sin 72°}=1.97\text{ cm}\ (3\text{ s.f.})$$

10 a In triangle ABD:

$$\frac{\sin B}{b}=\frac{\sin A}{a}$$

$$\frac{\sin B}{76}=\frac{\sin 66°}{136}$$

So $\sin B=\frac{76\sin 66°}{136}$

$B=30.7°$ (3 s.f.)

10 a So the angle between AB and BD is 30.7°.
Using triangle BCD:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{80} = \frac{\sin 98°}{136}$$

So $\sin B = \dfrac{80 \sin 98°}{136}$

$B = 35.6°$ (3 s.f.)

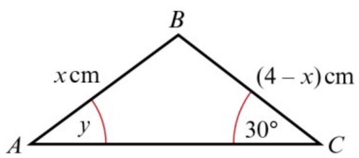
So the angle between BC and BD is 35.6°.
The angle between the fences AB and BC is 30.7° + 35.6° = 66.3°.

b In triangle ABD:
Angle ADB = 180° − 66° − 30.7° = 83.3°
Using the cosine rule:

$$d^2 = a^2 + b^2 - 2ab\cos D$$
$$= 136^2 + 76^2 - 2 \times 136 \times 76 \cos 83.3°$$
$$= 18\,496 + 5776 - 2411.817\,477$$
$$= 21\,860.182\,52$$

So d = 147.851...
The length of the fence AB is 148 m (3 s.f.).

11



Using $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$$\Rightarrow \frac{4-x}{\sin y} = \frac{x}{\sin 30°}$$

$$\Rightarrow (4-x)\sin 30° = x \sin y$$

$$\Rightarrow (4-x) \times \frac{1}{2} = x \times \frac{1}{\sqrt{2}}$$

Multiply throughout by 2:

$$4 - x = x\sqrt{2}$$
$$x + \sqrt{2}x = 4$$
$$x\left(1+\sqrt{2}\right) = 4$$
$$x = \frac{4}{1+\sqrt{2}}$$

11 Multiply 'top and bottom' by $\sqrt{2} - 1$:

$$x = \frac{4\left(\sqrt{2}-1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}$$
$$= \frac{4\left(\sqrt{2}-1\right)}{2-1}$$
$$= 4\left(\sqrt{2}-1\right)$$

12 a Using the left-hand triangle, the angles are 40°, 128° and 12°. (128° = 180° − 52° and 180° − (128° + 40°) = 12°)

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 128°} = \frac{15}{\sin 12°}$$
$$a = \frac{15 \sin 128°}{\sin 12°}$$
$$a = 56.8518\ldots$$

Using the larger right-angled triangle:

$$\sin 40° = \frac{\text{height}}{56.8518}$$

Height = 56.8518 sin 40° = 36.54...
The height of the building is 36.5 m (3 s.f.).

b Assume that the angles of elevation have been measured from ground level.

